Questions 1, 2 and 3 each weigh 1/3. These weights, however, are only indicative for the overall evaluation.

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MONETARY POLICY
SUGGESTED SOLUTIONS TO AUGUST 8 EXAM, 2018

QUESTION 1:

Evaluate whether the following statements are true or false. Explain your answers.

(i) Consider a dynamic general equilibrium model where infinitely lived individuals have per-period utility functions $u(c_t, m_t, l_t)$, where c_t is consumption, m_t denotes real money balances, l_t denotes leisure, and where output y_t is produced according to $y_t = f(k_{t-1}, 1 - l_t)$, where k_{t-1} is the capital stock. In this setting money is always superneutral.

A False. To explain this, it suffices to look at the steady state. From the consumption Euler equation $u_c(c_t, m_t, l_t) = \beta(1 + r_t) u_c(c_{t+1}, m_{t+1}, l_{t+1})$ we have the steady-state relationship $1+r = 1/\beta$. As the real interest rate equals the marginal product of capital, we have the condition $1 + f_k(k^{ss}, 1 - l^{ss}) = 1/\beta$. In the case of exogenous labour supply, we would have long run superneutrality. However, as labour supply is endogenous, and characterized by the optimality condition guiding the consumption-leisure choice, $u_l(c^{ss}, m^{ss}, l^{ss}) = -u_c(c^{ss}, m^{ss}, l^{ss}) f_l(k^{ss}, 1 - l^{ss})$, superneutrality only holds in the steady state, if both $u_l(c^{ss}, m^{ss}, l^{ss})$ and $u_c(c^{ss}, m^{ss}, l^{ss})$ are independent of m^{ss} (e.g., if utility is separable in all arguments). Otherwise superneutrality fails in directions depending upon the cross derivatives of the utility function.[1]

[1] A description of the various possibilities is not necessary.

(ii) Assume that empirical analyses show that country A follows a monetary policy rule $i_A = 1.5\pi_A$, and country B follows $i_B = 1.5\pi_B + 0.5x_B$, where i, π and x denote, respectively, the nominal interest rate, the inflation rate and the output gap. Subscripts distinguish countries. From these analyses, it follows that country B has preference for output gap stability while country A has not.

A False. The coefficients in an estimated interest rate rule does not necessarily say anything about preferences. They may as well reflect different policies for identical preferences because economic structures differ. In the example, country A and B could share identical preferences for an exclusive focus on inflation stability, but in country B, the output gap has proven to be a good indicator for inflation (in contrast with country A). Hence a positive response to x_B does not necessarily reveal a preference for output stability *per se*.

(iii) In the simple New-Keynesian model with monopolistic competition and sticky prices, a monetary policy implementing the Friedman rule is optimal as it eliminates any relative demand distortions.

A False. In the New-Keynesian model with sticky prices, inflation causes relative price changes as some firms can change their price, while others cannot. Therefore, a monetary policy implementing the Friedman rule will not be optimal as deflation would also cause relative price changes. Only zero inflation is optimal.

QUESTION 2:

Consider the following static, log-linear IS/LM-style model:

$$y = -\alpha i + u, \qquad \alpha > 0 \tag{1}$$

$$m = -ci + y + v, \qquad c > 0, \tag{2}$$

where y is output, i is the nominal interest rate (in deviations from some steady state), m is the nominal money supply, and u and v are mean-zero, independent shocks with variances σ_u^2 and σ_v^2, respectively. The objective of monetary policy is to minimize output variance, and policy is conducted before the shocks hit the economy.

(i) Discuss briefly (1) and (2), and derive optimal monetary policy when m is the policy instrument and when i is the instrument. Then show that i is the preferable instrument when

$$\left(1 + \frac{2c}{\alpha}\right)\sigma_u^2 < \sigma_v^2. \tag{3}$$

Provide the intuition for condition (3) with particular focus on the variance terms.

A Equations (1) and (2) are IS and LM curves, respectively, in a fixed price model. Aggregate demand decreases with the nominal interest rate, as does money demand. When m is the policy instrument, y can be solved by (1) and (2) as a function of m:

$$y = \frac{\alpha m + cu - \alpha v}{\alpha + c}$$

As the shocks are unknown when setting m, the output-variance minimizing policy is $m = 0$. When the interest rate is the instrument, the IS curve gives output immediately as

$$y = -\alpha i + u$$

and optimal policy follows as $i = 0$.

With optimal policy under money supply as an instrument, $m = 0$, output variance is

$$\mathrm{E}_m [y_t]^2 = \frac{c^2\sigma_u^2 + \alpha^2\sigma_v^2}{(\alpha + c)^2}.$$

With optimal policy when the interest rate is the instrument, $i = 0$ leads to

$$\mathrm{E}_i [y_t]^2 = \sigma_u^2.$$

The interest rate is thus the preferred instrument if the output variance is lowest under that policy procedure.

This requires

$$\begin{aligned} \mathrm{E}_i [y_t]^2 &< \mathrm{E}_m [y_t]^2, \\ \sigma_u^2 &< \frac{c^2\sigma_u^2 + \alpha^2\sigma_v^2}{(\alpha + c)^2}, \\ \sigma_u^2 \left[(\alpha + c)^2 - c^2\right] &< \alpha^2\sigma_v^2, \\ \sigma_u^2 \left[\alpha (\alpha + 2c)\right] &< \alpha^2\sigma_v^2, \end{aligned}$$

and thus

$$\left(1 + \frac{2c}{\alpha}\right)\sigma_u^2 < \sigma_v^2.$$

The condition shows that high money market volatility (σ_v^2) relative to high goods demand volatility (σ_u^2) is conducive for choosing the nominal interest rate as instrument. In effect, the economy will be perfectly insulated from money market shocks. On the other hand, if demand shocks are predominant, the inequality will fail, as having the interest rate endogenously increase when a positive goods demand occurs, will have a stabilizing effect not present under an interest rate targeting procedure.

(ii) Consider an extension where

$$m = b + hi + \omega, \qquad h > 0, \tag{4}$$

is added to the model. In equation (4), b is the money base, which is now a possible monetary policy instrument, and ω is a mean-zero shock with variance σ_ω^2. The variable m is now interpreted as an endogenous broad measure of money. Is the case for using a nominal interest rate operating procedure strengthened or weakened relative to condition (3) in this extended version of the model? A thorough verbal discussion is sufficient.

A The case is strengthened as the introduction of the shock ω essentially increases economic volatility stemming from the money market. One can show that under a monetary base operating procedure it is optimal to set $b = 0$, which yields output as:

$$y_t = \frac{(c+h)\, u_t - \alpha v_t + \alpha \omega_t}{\alpha + c + h}.$$

Output variance becomes

$$\mathrm{E}_b \left[y_t\right]^2 = \frac{(c+h)^2 \sigma_u^2 + \alpha^2 \sigma_v^2 + \alpha^2 \sigma_\omega^2}{(\alpha + c + h)^2},$$

emphasizing the additional volatility induced by σ_ω^2. On the other hand, the solution under an interest rate operating procedure is unchanged.

(iii) Assume that monetary policymaking takes the form of a money base rule of the form $b = \mu i$. If there are no shocks to the monetary side of the model, $\sigma_v^2 = \sigma_\omega^2 = 0$, will a "pure" money base rule, $\mu = 0$, be optimal? Explain.

A The answer is no. Even though there are no shocks from the monetary side of the model, it is not optimal to chose a constant b. In that case, a positive goods demand shock would increase output and the nominal interest rate in the conventional IS/LM fashion. The endogenous increase in the interest rate automatically stabilizes some of the impact of the shock. But letting $\mu < 0$, b falls in response to the increasing nominal rate, thereby further dampening the increase in output.

QUESTION 3:

Consider the following log-linear model of a closed economy:

$$x_t = \mathrm{E}_t x_{t+1} - \sigma^{-1}\left(i_t - \mathrm{E}_t \pi_{t+1} - \rho - r_t^n\right), \qquad \sigma > 0, \qquad \rho > 0 \tag{1}$$

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t, \qquad 0 < \beta < 1, \quad \kappa > 0, \tag{2}$$

where x_t is the output gap, i_t is the nominal interest rate (the monetary policy instrument), π_t is goods price inflation and $r_t^n \equiv \rho + e_t$ is the natural rate of interest, with e_t assumed to be a mean-zero, serially uncorrelated shock. E_t is the rational expectations operator conditional on all information up to and including period t.

(i) Discuss the micro-economic foundations behind equations (1) and (2).

A Here is should be mentioned that (1), a dynamic IS curve, is derived from a log-linearization of consumers' consumption-Euler equations: A higher real interest rate, $i_t -$ $\mathrm{E}_t \pi_{t+1}$ relative to the steady-state value ρ, make consumers increase future consumption relative to current. Equation (2), a New-Keynesian Phillips Curve, is derived from the optimal price-setting decisions of monopolistically competitive firms that operate under price stickiness. Prices are set as a markup over marginal costs, and as the output gap is proportional to marginal costs, it enters (2) positively. The more price rigidity (e.g., the lower a probability of price adjustment under a Calvo price-setting scheme), the smaller is κ. Expected future prices are central for price determination, as firms are forward looking, since they acknowledge that the price set today may be effective for some periods.

(ii) Assume that the monetary authority minimizes the loss function

$$L = \frac{1}{2} \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\lambda x_t^2 + \pi_t^2\right], \qquad \lambda > 0. \tag{3}$$

Discuss the micro-economic foundations for this loss function.

A This type of loss function can be derived as the second-order Taylor approximation to (the negative of) the representative household's utility function. Price rigidity causes losses from aggregate mark-ups being different from the desired markup, and under the Calvo-price structure, staggering cause inefficient dispersion of consumption of various goods. The quadratic terms in (3) reflect the costs from these fluctuations. Inflation is proportional to the inefficient goods dispersion, and output gap fluctuations are proportional to the fluctuations in the mark-up (that causes inefficient fluctuations in consumption and labor).

(iii) Derive the optimal values of x_t and π_t under discretionary policymaking [Hint: Consider x_t the policy instrument, and acknowledge that under discretion the optimization problem becomes a sequence of static problems as expected values can be taken as given]. Discuss the solutions, and describe how the nominal interest rate will move with the natural rate of interest in equilibrium.

A Using the hint, the first-order condition is found as

$$\lambda x_t + \kappa \pi_t = 0.$$

This is inserted into (2) in order to substitute out x_t:

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} - \left(\kappa^2/\lambda\right) \pi_t$$

which yields

$$\pi_t = \left[\beta/\left(1+\kappa^2/\lambda\right)\right] \mathrm{E}_t \pi_{t+1}.$$

Since the characteristic root of this difference equation is unstable $\left(\left(1+\kappa^2/\lambda\right)/\beta > 1\right)$, the unique non explosive solution for inflation is $\pi_t = 0$ all t. In combination with the first-order condition, one recovers $x_t = 0$ as well. Hence, under discretionary policymaking, inflation and the output gap is fully stabilized (leading to the lowest possible welfare loss). In terms of the actual policy instrument, the nominal interest rate, it follows from (1) that it will be given by $i_t = e_t$. Hence, the policy rate is constantly adjusted so as to follow the natural rate of interest (here, ignoring a lower bound on the nominal interest rate). In consequence, demand is adjusted to equal flex-price output at all times. Actual output follows the natural rate of output leaving the output *gap* unchanged.